MAT 1700

LØSNINGSFORSLAG

SEMINAR #7

Seminar #7

Oppgave 1 (Ref. oppgave 6 - Seminar #6)

$U(x,y) = xy$
$p_x = 9$ (initially)
$p_y = 1$ (composite good)
$m = 72$

Consumption max. utility basket:

$$MU_x / MU_y = \frac{y}{x} = \frac{p_x}{p_y} = \frac{9}{1}; \; y = 9x$$

and: $9x + y = 72 \Rightarrow 9x + 9x = 72; \; \underline{\underline{x = 4}}$, $\underline{\underline{y = 36}}$

(a) **Kompensasjonsmargin** (compensating variation ≡ CV) and **equivalensmargin** (equivalent variation ≡ EV)

CV: $U(x,y) = \underline{144} = 4 \cdot 36$ (kept unchanged)

'New' price; $p_x = 4$

(1) $4x + y = 72$, and

(2) $y = 4x$

Thus; $U(x,y) = x(4x) = 144; \; \underline{\underline{x = 6}}, \; y = \underline{\underline{144/6 = 24}}$

"Decomposition - basket"

Budget line: $p_x \cdot x + p_y \cdot y = m$; ie.

$4(6) + 1(24) = \underline{48}$

$\underline{\underline{CV}} = (m - \text{budget}|_{CV}) = 72 - 48 = \underline{\underline{24}}$ (See interpretation in sol. manual seminar #6)

EV: At final consumption-basket (when p_x reduced to 4 from 9)

$$\left.\begin{aligned} 4x + y &= 72 \\ y &= 4x \end{aligned}\right\} \Rightarrow \underline{\underline{x = 9}}; \; \underline{\underline{y = 36}}$$

$U(x,y) = U(9,36) = 9 \cdot 36 = 324$ evaluated at $p_x = \underline{\underline{9}}$ and 'decomposition-basket' ($x = 6$, $y = 24$);

$U(x,y) = x(9x) = 324; \; x = 6; \; y = 54$

Budget $= 9(6) + 1(54) = 108 > m = 72$

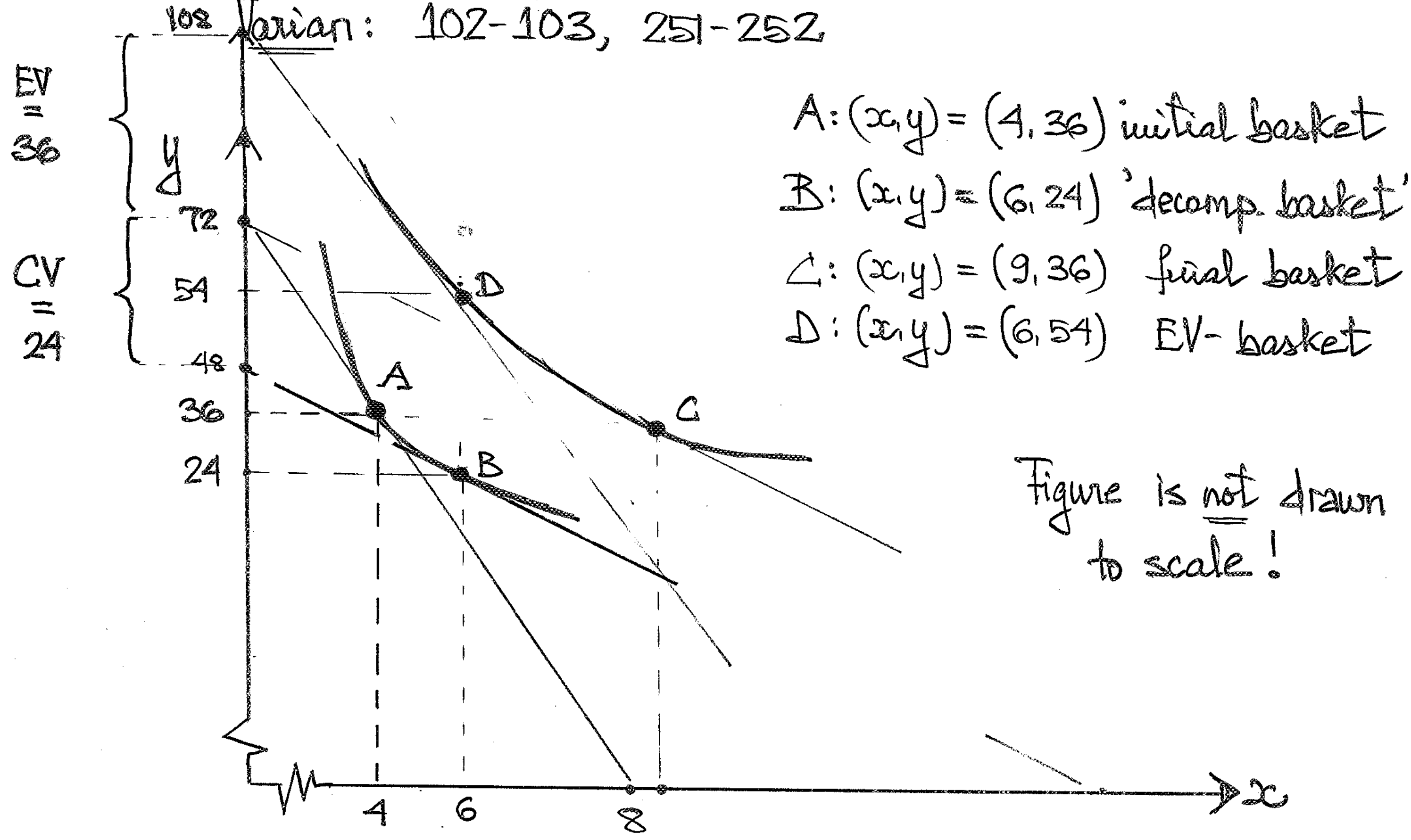
$\underline{\underline{EV}} = 108 - 72 = \underline{\underline{36}}$

Oppgave 1, fortsettelse

(b) CV ≠ EV.... hvorfor?

Jfr. løsn.-forslag oppgave 6 - seminar #6.

Varian: 102-103, 251-252



B ≡ CV-basket = (6, 24) ⇒ CV = 72 − 48 = 24

D ≡ EV-basket = (6, 54) ⇒ EV = 108 − 72 = 36

Oppgave 2 Etterspørsel pris-elastisitet

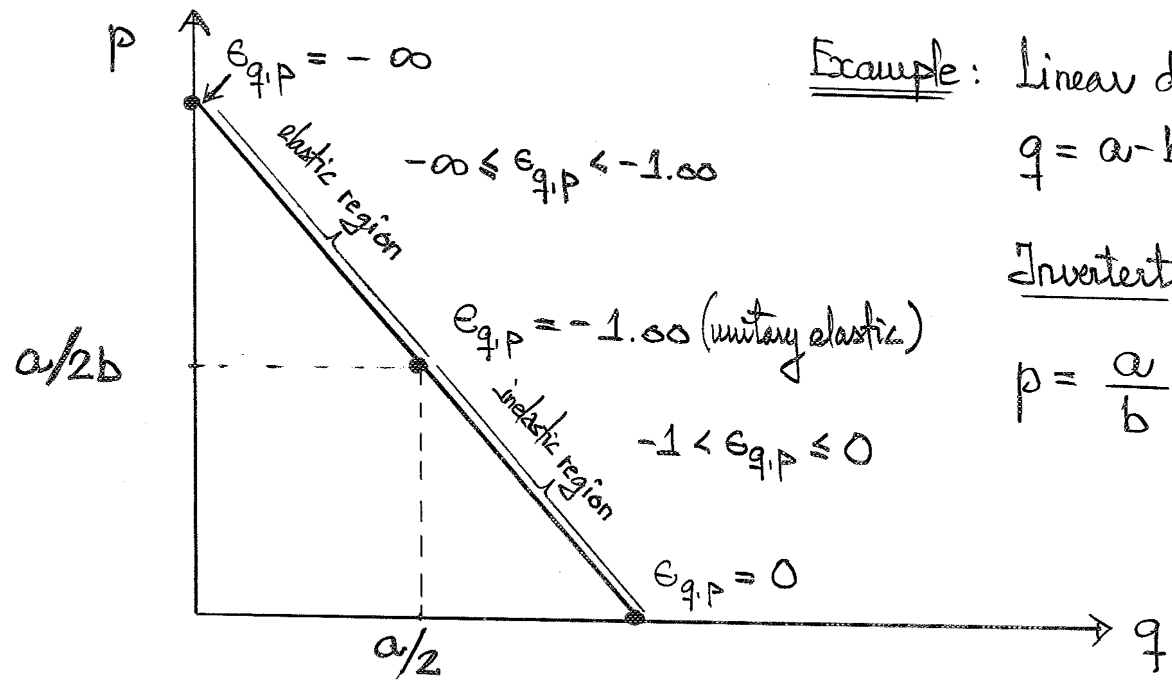
$\Delta p = 5{,}75 - 5{,}00 = \underline{0{,}75}$

$\Delta q = 800 - 1000 = -\underline{200}$

Varian, pp. 270 – 273 "Elasticity of demand"

$$\epsilon_{q,p} = \frac{p}{q} \cdot \frac{\Delta q}{\Delta p} = \frac{5{,}00}{1000}\left(\frac{-200}{0{,}75}\right) = \underline{\underline{-1{,}33}}$$

"% Δ in quantity divided by % Δ in price"



Example: Linear demand

$q = a - bp$; $a, b > 0$ 'constants'

Invertert demand-kurve;

$$p = \frac{a}{b} - \frac{1}{b} q$$

Fordi $\Delta q / \Delta p \equiv$ slope of demand curve $= -b$; then we have

$$\epsilon_{q,p} = \frac{p}{q}(-b) = \frac{-bp}{a - bp} = -b\left(\frac{p}{q}\right)$$

"% Δ in quantity brought about by a one percent change in price"

$$\epsilon_{q,p} = \underline{-1} \Rightarrow \underline{p = a/2b}; \ \underline{q = a/2}$$

Merk: Slope of demand-ftn. $= -b$ is constant throughout, whereas price-elasticity of demand $= -bp/q$ varies along the demand-curve!

Slope-coefficient $= -b$ measures absolute change in quantity (measured in units) caused by a one unit change in price!

Oppgave 3 Price-elasticity of demand

$$Q = 100 - 2P; \quad P = 100/2 - \tfrac{1}{2}Q = 50 - \tfrac{1}{2}Q$$

(a) Etterspørsel i kroner; $Q \times P$ = quantity × price

Pris	Quantity	Pris × Quantity
0	100	0
10	80	800
$\boxed{20}$	60	$\boxed{1200}$
$\boxed{25}$	50	$\boxed{1250}$
$\boxed{30}$	40	$\boxed{1200}$
40	20	800
50	0	0

Merk: For $a/2 = q = \frac{100}{2} = 50$ er $p = \frac{100}{2(2)} = 25$; koordinatene (50, 25) definerer punktet på etterspørselskurven hvor pris-etterspørselen er enhets (unitary) elastisk.

(b) $\epsilon_{q,p} = -1 = \dfrac{-2(25)}{100-2(25)} = \dfrac{-bp}{a-bp} = \dfrac{-bp}{q}$ "unitary elastic".

$|\epsilon_{q,p}| > 1$ E.g. $p = 30 > 100/2(2)$ (Price-decrease ⇒ increased revenue!)

$|\epsilon_{q,p}| = \dfrac{|-2(30)|}{|100-2(30)|} = \dfrac{|60|}{|40|} = |1{,}50|$

Elastic region; see table above

(See Varian, figure 15.4 (page 272)) ... a price-decrease (in this region of the demand-curve) increases demand by 50%

E.g: price ↓ (from 40 → 30) ⇒ − 25%

demand ↑ (from 800 → 1200) ⇒ + 50%

$|\epsilon_{q,p}| < 1$ E.g. $p = 20 < 100/4$

$= \dfrac{|-2(20)|}{|100-2(20)|} = \dfrac{|40|}{|60|} = |0{,}67|$

Inelastic region of the demand curve ⇓ price-decrease causes decrease in revenue (= p×q)

E.g: price ↓ (from 20 → 10) ⇒ −50% decrease

demand ↓ (from 1200 → 800) ⇒ −33% decrease

Oppgave 3, fortsettelse

(a) $Q = 1200/P$ .... pris-elastisiteten ?

$(p \times q) \equiv$ tot. demand (revenue) $=$ 1200 (constant)

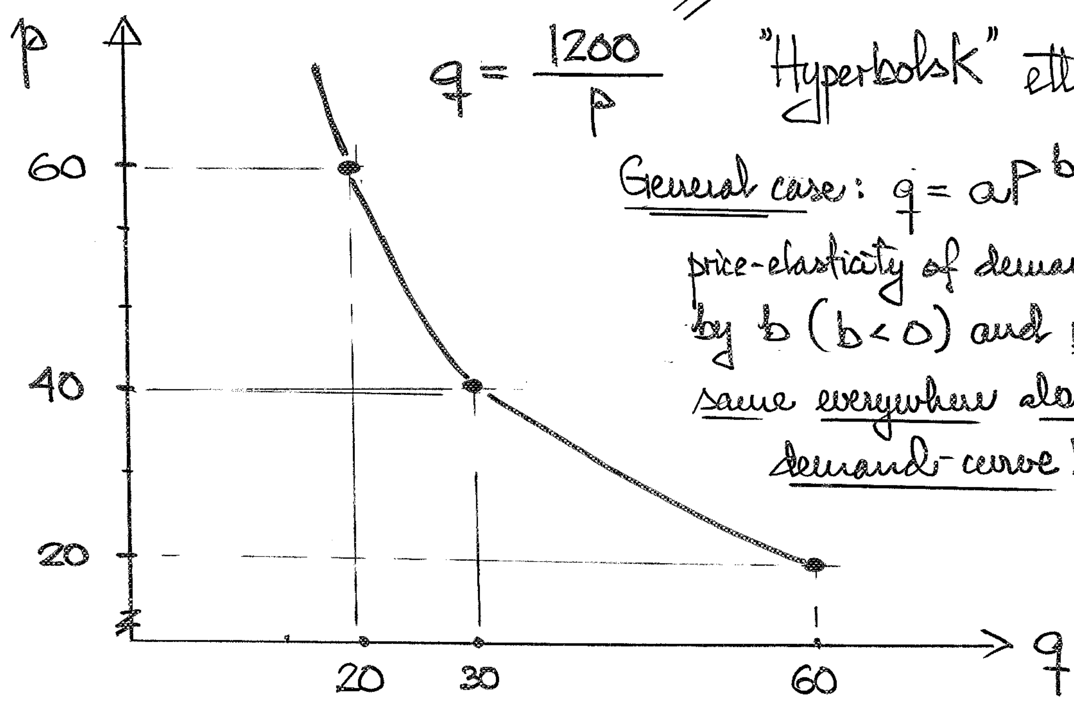
ie. demand = 1200 regardless of price = p along the demand-curve (see below).

Therefore; |price-elasticity of demand| = 1.00 "unitary elastic" everywhere along this demand-curve; ie. no need to be specific about the point at which elasticity is to be measured!

price	quantity
60	20
40	30
20	60

$q = 1200p^{-1} = aP^b$

$q = \frac{1200}{p}$ "Hyperbolsk" etterspørsel



General case: $q = aP^b$ $(b < 0)$ price-elasticity of demand is given by b $(b<0)$ and remains the same everywhere along such a demand-curve!

Oppgave 4 Prod. technology

$$Q = 50\sqrt{M \cdot L} + M + L$$

(a) Skalaavkastningen ('returns to scale, RTS')

Antar at innsatsfaktorene økes med en (konstant) faktor lik λ .... øker output (Q) med samme faktor, mer enn λ eller mindre enn λ?

$$Q_\lambda = 50\sqrt{\lambda M \lambda L} + \lambda M + \lambda L$$

$$= 50\lambda\sqrt{M \cdot L} + \lambda M + \lambda L = \lambda\left[50\sqrt{M \cdot L} + M + L\right]$$

$Q_\lambda = \lambda \cdot Q$ ... increasing inputs by λ, output increases by λ. Thus, the prod. ftn Q exhibits constant returns to scale.

(b) $MP_L = \frac{1}{2}(50)\sqrt{M}/\sqrt{L} + 1.00 = \dfrac{25\sqrt{M}}{\sqrt{L}} + 1.00$

Antar at $M > 0$ and a constant when analysing this question: Increasing L has the effect of decreasing MP_L; the MP_L decreases for all levels of L, but the $MP_L \geq 1.00$

Oppgave 5 Prod. technology

Skalaavkastning = returns to scale (RTS)

$$Q_1 = A \cdot L_1^{\alpha} K_1^{\beta}$$

$\bar{A}, \bar{\alpha}, \bar{\beta} > 0$ (positive konstanter)

$$\text{La } Q_2 = A(\lambda L_1)^{\alpha}(\lambda K_1)^{\beta} = A\lambda^{\alpha} L_1^{\alpha} \lambda^{\beta} K_1^{\beta}$$

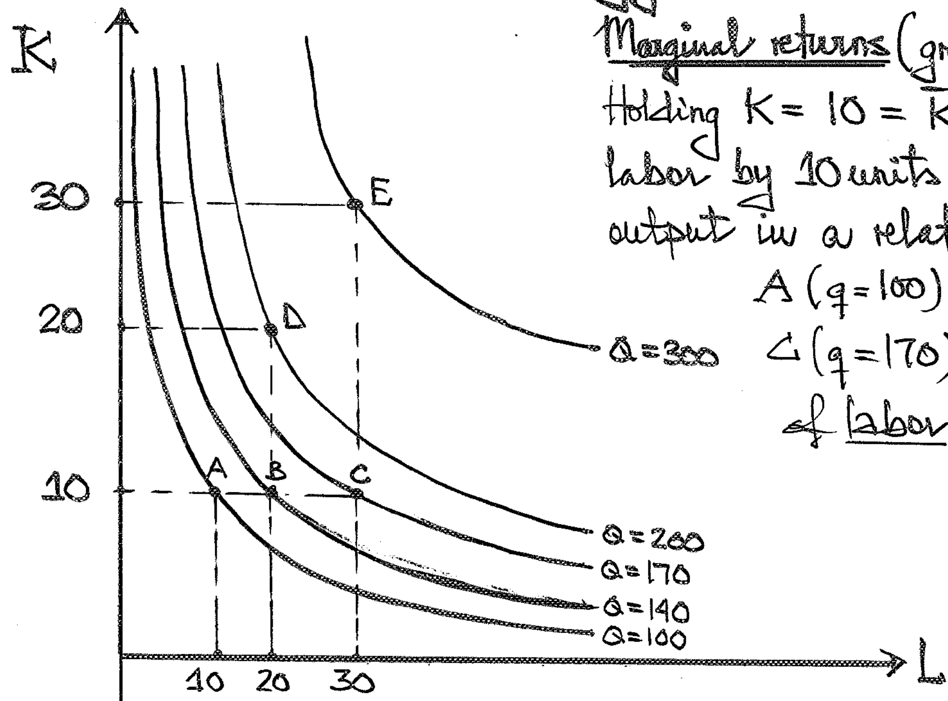
$$Q_2 = \lambda^{(\alpha+\beta)} A L_1^{\alpha} K_1^{\beta} = \underline{\underline{\lambda^{(\alpha+\beta)} Q_1}}$$

(i) økende skalavkastning $\Rightarrow (\alpha+\beta) > 1.00$; then
$\lambda^{(\alpha+\beta)} > \lambda \Rightarrow Q_2 > Q_1$ (increasing RTS)

(ii) konstant skalaavkastning $\Rightarrow (\alpha+\beta) = 1$; then $Q_2 = Q_1$

(iii) avtagende skalavkastning $\Rightarrow (\alpha+\beta) < 1$; then $Q_2 < Q_1$

Oppgave 6 Prod. technology $Q = 10 L^{1/2} K^{1/2}$



Marginal returns (grenseavkastning): Holding $K = 10 = \bar{K}$ (fixed); increasing labor by 10 units successively decreases output in a relative sense; from A ($q=100$), to B ($q=140$) to C ($q=170$) ie. marginal product of labor is diminishing (avtakende).

Returns to scale (RTS): Doubling inputs (by a factor of 2) (from pt. A to pt. D) doubles output ($q = 200$)
Tripling inputs (by factor of 3) triples output ($q = 300$)
ie. constant returns to scale

Turn! %

Oppgave 6, fortsettelse

$$Q_1 = 10 L^{1/2} K^{1/2}$$

$$Q_2 = 10 \lambda^{1/2} L^{1/2} \lambda^{1/2} K^{1/2} = \lambda^{(1/2+1/2)} 10 L^{1/2} K^{1/2}$$

$$Q_2 = \lambda^1 \cdot Q_1 \equiv Q_\lambda$$

ie. constant RTS (ref. løsn.forslag oppgave 5 her)